Fluid Mechanics
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Lecture - 06
Buoyancy, Metacentre, Stability and Rigid Body Motion

Welcome you to the lecture six on fluid mechanics. In the last class we discussed about fluid statics that means fluid at the rest. And again, I want to repeat it that I have been following these three reference books. One is Cengel Cimbala, Fluid Mechanics Fundamentals and the Applications; they very concise book on Frank M. White that is the Fluid Mechanics; and the Fluid Mechanics by Bidya Sagar Panis.

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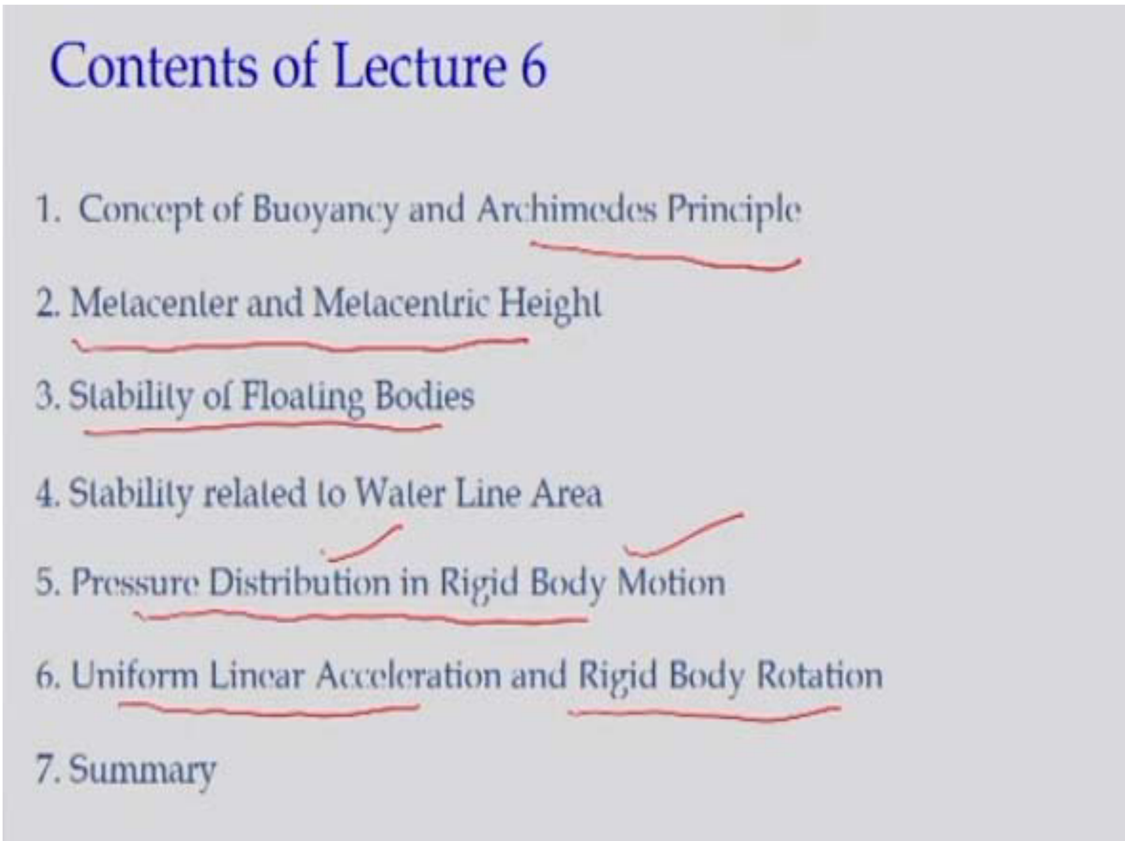

Now let us go to the next levels. We will discuss today the concept of the buoyancy, very well known Archimedes principles. In that we will also discuss it the metacenter or metacentric height how to determine the metacentric height of a floating object. And we also will talk about the stability of the floating object. And then we will talk about if there is a rigid body motions, the liquids in a rigid body motions.

What could be the pressure diagrams and also we will discuss is that rigid body motions when you have a uniform angular rotations.

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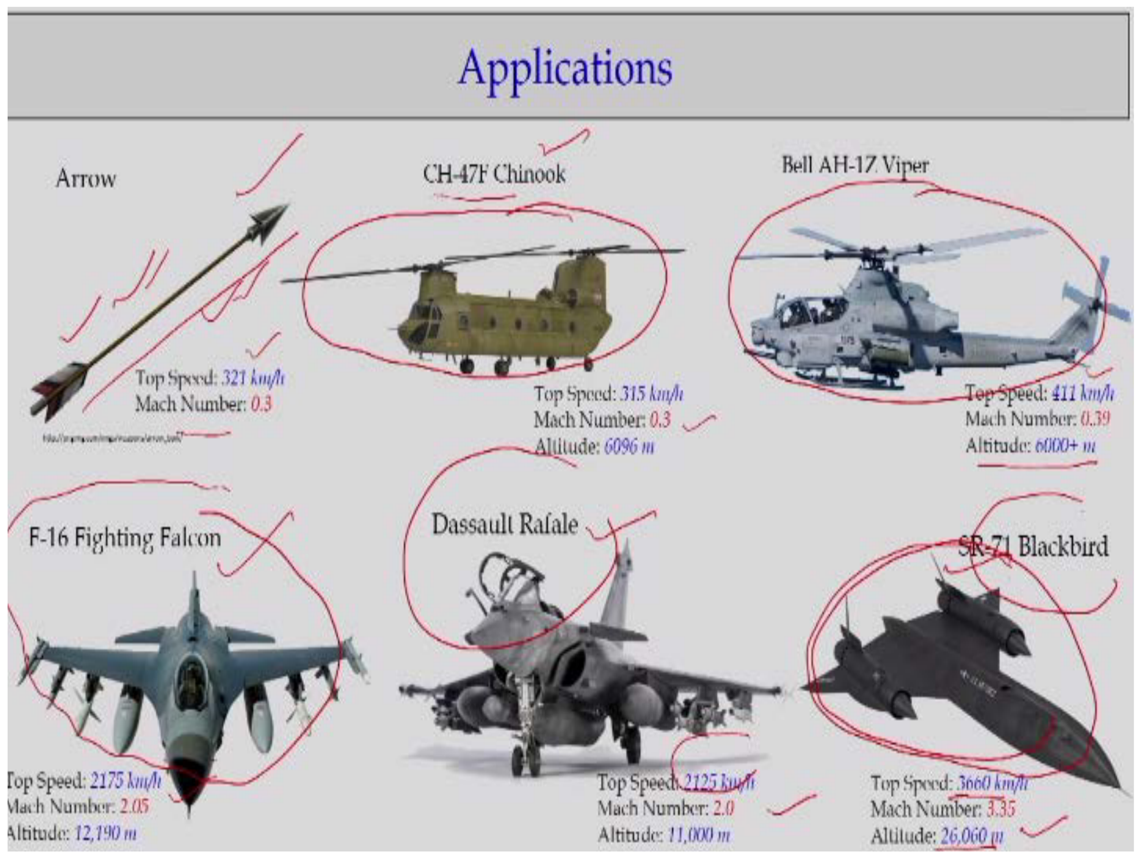

With this, let we start the lectures today. Before starting these lectures, let us see this slides. This is what is showing that how the fluid mechanics has changed our life. If you look at this arrow which is maybe very ancient times first the human civilization, starting the hunting. Hunting is a main occupations for the life. That is what having a top speed of 321 kilometer per hour and as equivalent to Mach number of 0.3.

So the art or knowledge of aerodynamics of what I would to say it that these the arrow or the aerodynamics of the arrows, that is what was known to very ancient people, just what they use for the hunting and the design of the arrows had changes as realized with the time, that different type of speed, different type of velocity, different type of range that is what is design.

And most of these designs which is done it from the experience. But what it happened maybe last one century we have developed fighter planes and that is what if we can look at the figures like the CH-47F Chinook with the top speed is 315 km/h which will have a Mach number of 0.3. So if you look it the arrangement of this, the aircraft.

Also you can see this arrangement of the aircraft and they are wings to create a vortex and because of creating the vortex that what will be difference the pressures and that will be resulting a uplift force. And the drag and the uplift force that what will give it two different vehicle this the difference vehicles which is having Mach numbers of 0.39, and it can go up to altitudes of 6000 meters and top speed is 411 kilometer per hours.

The similar way if you look at the newer flights what is the space flight or planes are available like the F-16, Rafales or the Blackbird. If you look at their speeds 2000 km/h and their Mach numbers is more than 2. And basically this Blackbird is having the top speed is close up to 3660 km/h and this altitudes can go up to 26,000. How this technology is developed so fastly to go to a Blackbird level because of the knowledge of fluid mechanics.

Not only the analytical fluid mechanics, the but also its develop in terms of numerical methods, the computational fluid dynamics, as well as the full scale the wind tunnel facility. The most of these advanced fighter planes are tested in full scale wind tunnel facilities.

So that is what my point to say it, if you look at the knowledge of the fluid mechanics can help us to design fighter aircraft, which can go as fast as 3600 km/h with a altitudes of 26,000 and which is the Mach numbers 3.35. So this what it is a possible because of the knowledge of the fluid mechanics.

So because of the extensive experiment conducting in full scale wind tunnels, series of numerical testing conducting for this type of the aircraft, it is now possible to develop this type of aircraft and not having any failure of this type of things. So what I am to emphasize is that the fluid mechanics knowledge is not limited to a text book of solving few academic problems. But we should have a more knowledge.

We should understand the fluid mechanics process more details as we are developing our fluid mechanics which are there more analytical way. Now the full scale wind tunnel facilities, the numerical methods CFD advancement of CFD made us to look the fluid mechanics in different way. So that is what my objective to say that fluid mechanics is now is a integral part to develop our the aircraft systems what I have just given you the examples.

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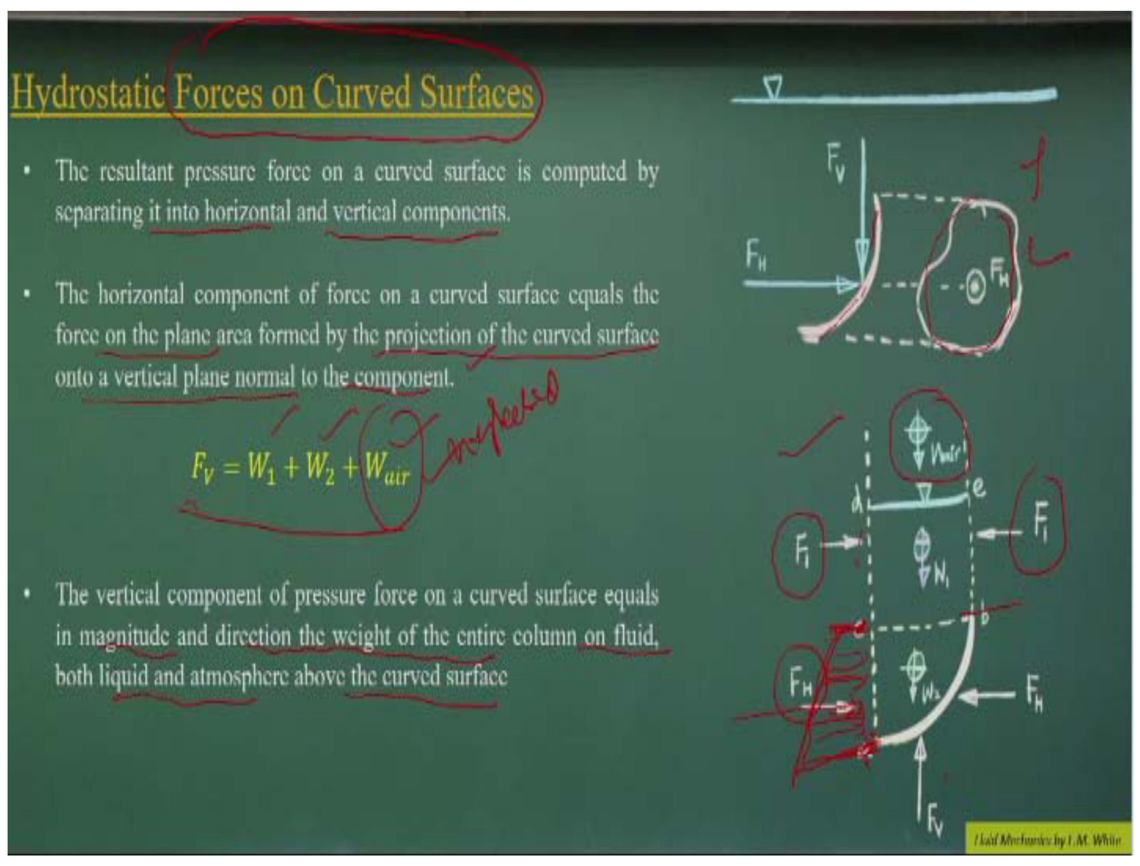

Let us come back to the topics what we have been discussing that fluid at the rest okay. As I told it when the fluid is the rest we have the two force component. One is force due to the pressure and the second is the gravity force. There is no velocity components, there is no shear stress component. So that is the reasons if you take a control volumes, it is easy to draw the pressure diagrams and from that pressure diagrams, if you equate with the gravity force, you can solve the problems.

That is a very basic concept what we have used in fluid at the rest. So now if you look it if you have to determine what could be the forces on a curved surface? What the problems we face it. If you have a let me take this example. So you have the free surface and you have a curved surface and you have the liquid which is having the density let be the rho.

If it that conditions, what is the basic difference of this point is here that if you take any point as you know it the pressure is a scalar quantity and will have a the pressure component will be normal to the point where you were considering the surface. In that case, what it happens when you have a curved surface, the directions of the pressure that what changes from the point to point.

So because of that, we have to resolve this force component in three Cartesian coordinate x, y, z. Then we can take a small element, do the integrations over the surface because you know the pressure diagrams, then we can integrate it to solve get it what could be the force, because of this pressures and the weight of the fluid. That what can be done it.

The problems come it that we can simplify that ones like we can resolve the curved surface into a two-part horizontal and vertical components okay. Like here what the diagrams have shown it, it is a curved surfaces and that is a projected surface on the verticals and on that the horizontal force is acting it and verticals are in the downward directions.

If it that the component if I draw the free body diagram, there will be the free surface and the top of the free surface there will be weight due to the air. Then there will be weight above this curved surface W_1 and W_2. That is a weight of the liquid what is we have. Now if I draw this is my control volume and the pressure diagrams okay.

This is what my control volume and if I draw the pressure diagrams, I can draw the pressure diagrams, which will be the triangular shape. The linearly will increase this at z we go further down and down. So that way we will have the pressure diagram like this. So you can see that if you look it the point above the CV, the same pressure diagram in the both the side which is representing force F_1 here.

That means, these two force will be cancelled out when we take this total pressure diagram to find out what could be the force due to these pressures. So F_1, F_2 will be cancelled out. Only you will have the pressure component which will be the trapezoidal pressure component from this C point to this point. So in horizontal directions we will have the pressure that is what only the pressure diagrams will change over this curved surface will be a trapezoidal set.

So if this is the pressure diagrams, you know from the height, we can determine what is the pressure at the sea, what is the pressure this point. If I know it, the area of this triangles will show us what will be the force F H acting over that multiply with the area, we can get it what will be the pressure force is acting in the horizontal direction. In vertical direction, if we see these diagrams, the basically the weight of the liquid what will become as a vertical component.

$$F_V = W_1 + W_2 + W_{air}$$

That will be the F_v will be the balance by the weight of the liquid what we have and W_{air} can be neglected as compared to the W_1 and W_2. So it is very easy to compute the vertical component pressure force on curve equal to magnitude and direction of weight of the entire column on fluid both liquid and atmospheres above the curved surface. That what we have said that.

And what will be the horizontal force component that what will be force on the plane area that what is a projection of the curved surface on to a vertical plane normal to this component. That what either through the pressure diagrams you can compute the force in the horizontal directions or you get it what could be the projected area on these the vertical plane and find out its centroid locations.

At that centroid locations if you know the pressure multiply with area that what will we give you the force component. Then you know the line of action of the force where it is acting it. For the weight part, the vertical component as well as horizontal component. If you know these two force component in vertical and horizontal direction and their line of actions you can easily using the vector calculus algebra, you can compute what will be the resultant forces and where is the line of actions of resultant forces.

So we will solve the some numerical problems to demonstrate that. But very basic idea is that whenever you have a curved surface, you draw a pressure diagrams. Consider a control volume, draw the pressure diagrams. Then equate the force component in horizontal directions and the vertical direction and those force component you find out from the pressure diagram and the weight of the liquids.

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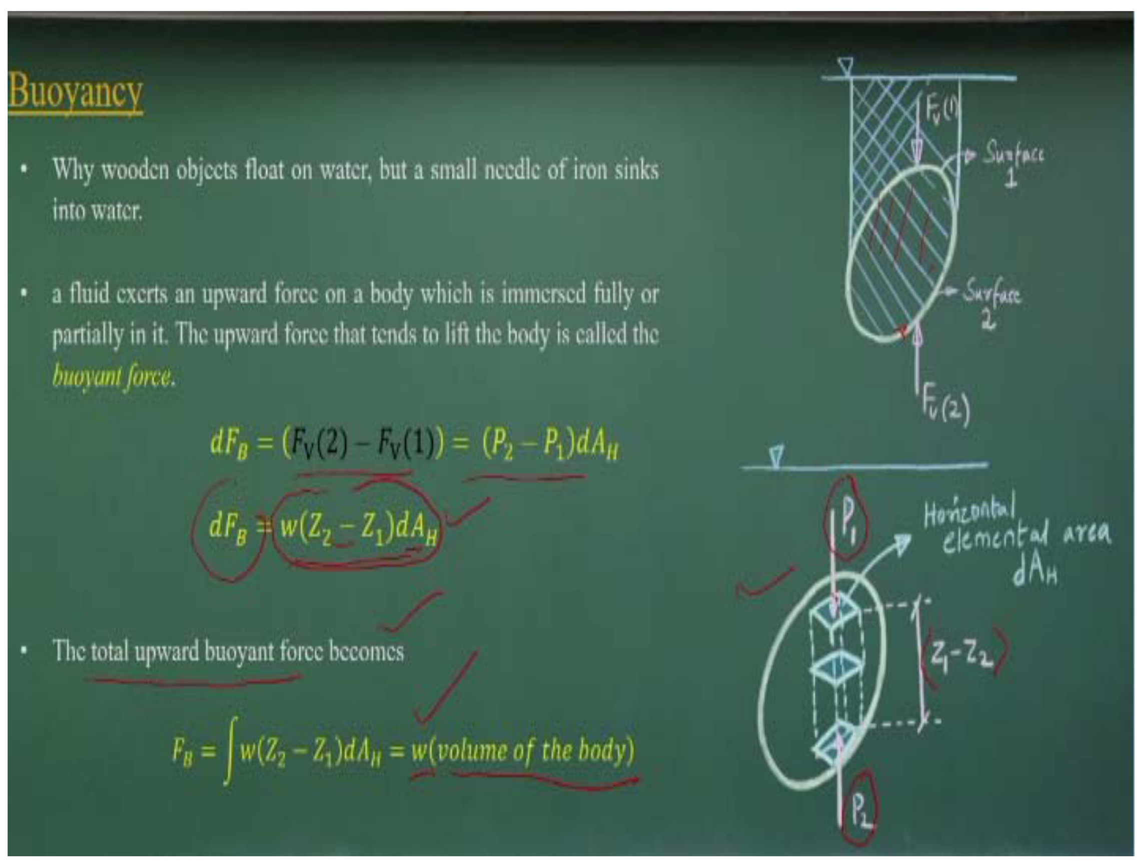

Now let us come it to very interesting topics what is we use in buoyancy as you know it the famous scientist Archimedes what invented this buoyancy concept when he was in a bathtub experienced the lighter weight because of the buoyancy forces. So what is the buoyancy force if you look it that, it is a very simple things that whenever you have any object, okay submerged within a liquid, what will happen it that there will be a force from the top also force from the bottom.

There will be force from the top because of this, the liquid mass, the top of the liquid mass that force vertical force will come it. And there will be force will be upward direction. How much that will come it. If you look it that, if take a control volumes like this, you have the surface where P_1 pressure is coming at the top, the P_2 pressure is coming to here.

And the distance between that these two point if we define Z_1 by Z_2 then you can find out pressure at the P_2 is a is equal to the height and the ρ g. Similar way the pressure at this point ρ g and the height from the free surface. So if you subtract the pressure multiplied with the dA_H, you can find out it will be the force component what will come it which is equivalent to weight of this element as equivalent to weight of liquid displayed by that control volume.

$$dF_B = (F_V(2) - F_V(1)) = (P_2 - P_1)dA_H$$

$$dF_B = w(Z_2 - Z_1)dA_H$$

That means this control volume what you have chosen it if you consider of submerged body that is as equal to the liquid that is that. That is what display the water, liquid because of the submergence of this object that what will be the vertical force what will get it, what will we experience it. So we will have a weight of the fluid. That is what is the unit weight and the volumes.

$$F_B = \int w(Z_2 - Z_1)dA_H = w(volume\ of\ the\ body)$$

The unit weight and the volume that what is surface area and the vertical height difference what we have. If you look at this total upward buoyant forces, if as I take in the element, if I do the integrations, finally we will get it unit weight of the liquid where we submerge the body and the volume of the body. That is what will be the buoyant force and what acts in upward directions, so that is what is a total upward buoyant force that what it works.

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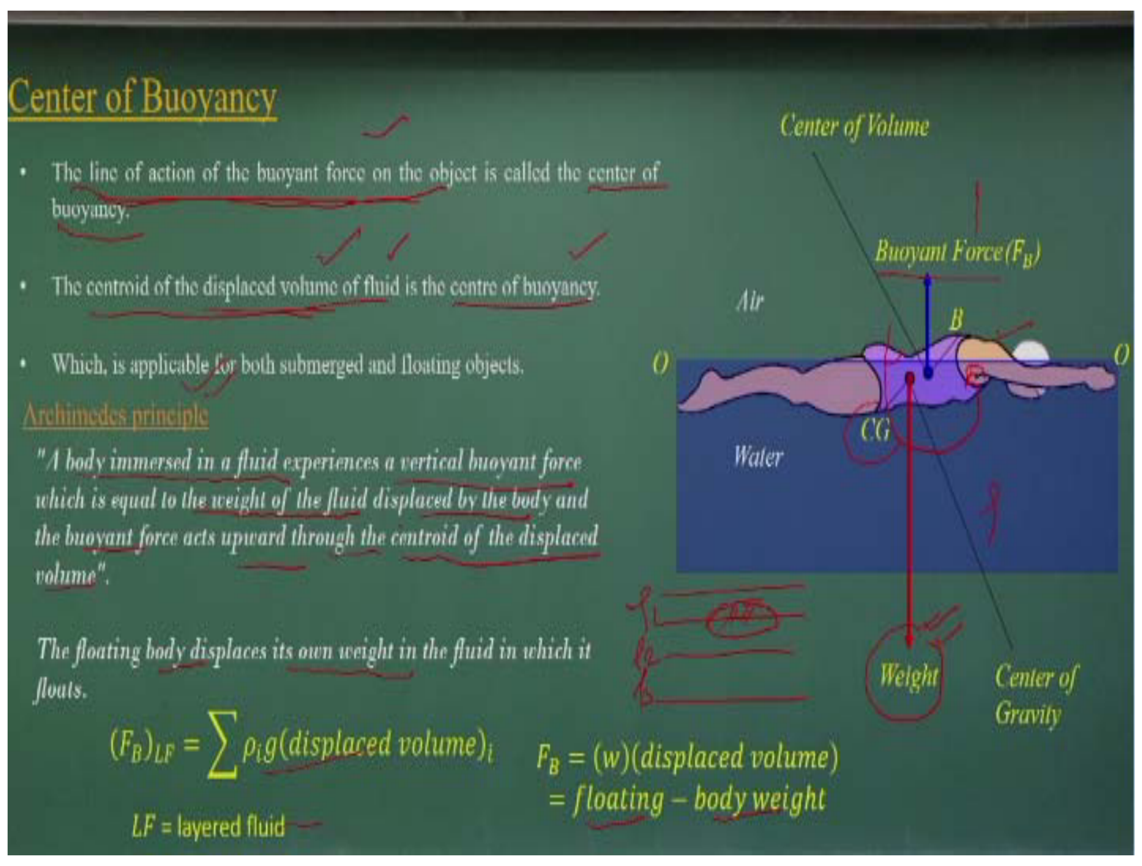

Now if you look it another point is we also look it that the center of buoyancy. That means the line of actions of the buoyant force like we try to compute for the other force component. So line of actions of the buoyant force is the center of buoyancy. That means at which point the buoyancy force acts. That is what the center of buoyancy.

That is what is necessary to counter with a gravity weight of the floating object. Like for example, if you look at the very interesting photographs what we have shown it here a swimmer, okay? You can see that the CG with the weight of the body is there, okay the center of gravity points and the CG weight of the body and the whatever the liquid displayed by this swimmer that what having a point here the center of buoyancy.

And at that center the buoyancy force is working like this. If you looking this figure you can say that there will be torque or moment will be because of these force in balance, okay. So that is the reasons if you look at any swimmers, they try to do some swimming activities to remain as a floating conditions. Because these the center of buoyancy and the CG they do not lie in the same locations.

Because of that there will be a torque will be there and that torque to balance it any swimmer if you look it to remain it floating conditions does slight bit swimming activities that change the flow pattern such a way that the additional force you can generate it to counter balance this moment. So very interesting studies like this if you look it that the basically the center of buoyancy if you look it that, that should be the centroid of displayed volume of the fluids.

So that is what to summarize and to make it very easy to say that the Archimedes principles what it says that a body immersed in a fluid experience a vertical buoyant force. That means whenever you immerse a body in a fluid, you will have a vertical buoyant force will act it which will be equal to weight of the fluid displayed by the body and the buoyant force act upward through the centroid of the displaced volume, which we discussed it.

Same thing is written in very concise way. That is what the Archimedes principle, okay. So is way back in two or three BC, okay. That is the knowledge of the fluid mechanics as old as just starting of the civilization. So if you look it that way, so we can say that the floating body displace the own weight. In case of the floating body what will happen it is that it can totally submerge in the floating body.

The floating body case, the weight of the object is counterbalanced by the buoyant force. So that what is happened. The buoyant force is counterbalanced by the weight of the object. So that is what it happens it, the floating and the body weight that what both will counterbalance it.

But some of the case you can have a layered fluid. That means you can have a different type of the fluids will be there with a different density like ρ_1, ρ_2 and ρ_3 and if you have a object is floating interface between two density the same concept we can consider it but you have to consider the pressure diagrams, you can find out which part of the volume is displayed by which liquid and based on that, you can have understand the problems will be the different as compared to single liquid concept where the ρ a single unit ρ value is there.

$$(F_B)_{LF} = \sum \rho_i g(displaced\ volume)_i$$

LF = layered fluid

$$F_B = (w)(displaced\ volume)$$

$$= floating - body\, weight$$

That is what is quite easier as compared to the layer fluids. Now let us discuss about the stability of floating object, stability of floating objects like the ship.

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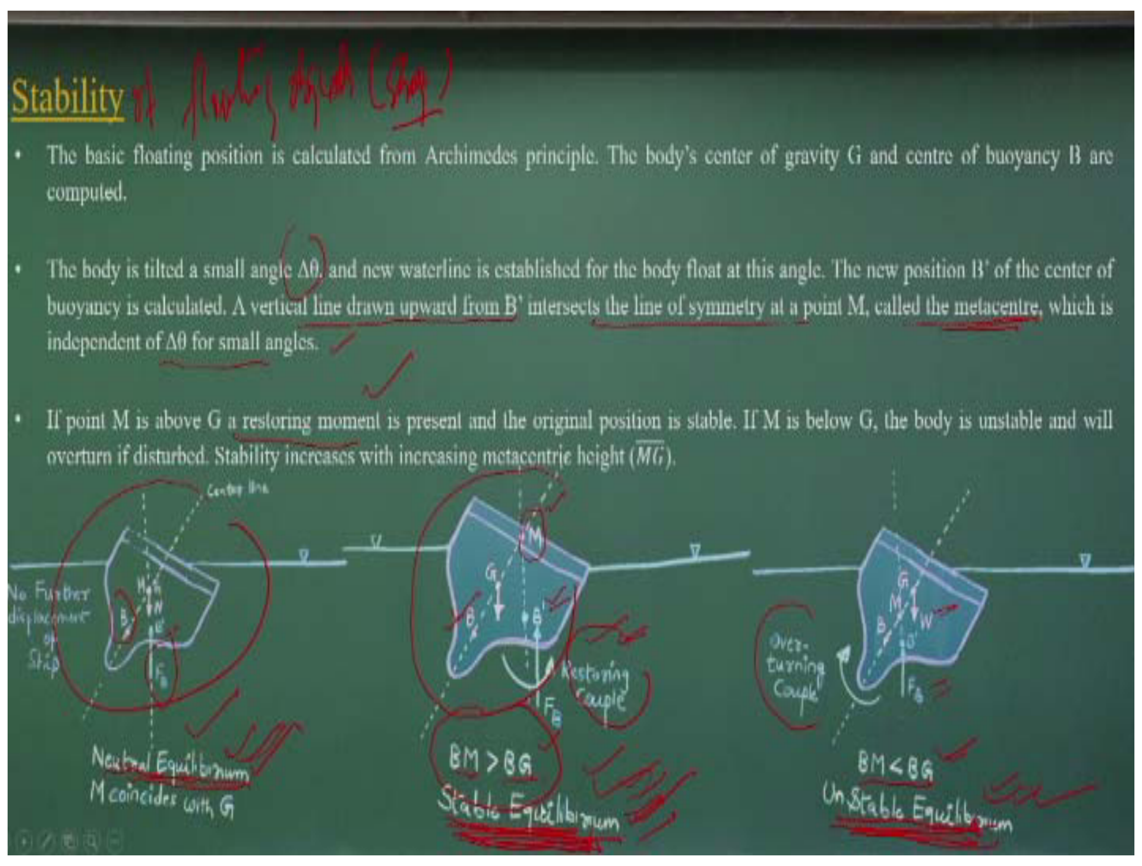

How much stability we have a floating objects. Now let us understand the principles the how the concept is conceived. As we discussed that there will be a gravity force which will act the CG of the floating object and there will be the force which is a buoyant force will act as the CG of the displaced liquid that what will be the center of buoyancy. So these two force components act at two different location.

One is a CG of floating object, another is a CG of the displaced volume of the fluid. Because of that, we will have the stability problems that if I just do a small disturbance or just slightly rotate the floating object, then what it happens it? Does that come back to original positions. If it is come back to original conditions, then I will tell it, it is a stable, it has a stability.

It can come back any small change will not change the stability of the floating object, it will come back to original position. But other way and others if I talk about unsteady unstable equilibrium. That means, if there is this slight bit change, slight bit tilting the floating objects and that what generate a large torque and finally capsized then we will tell it is the unstable equilibrium.

So let me discuss about this three equilibrium concepts. Natural equilibrium, stable equilibrium, and unstable equilibrium. So you can understand it if somebody wants to design a ship he has to find out the ship should have stable equilibrium conditions. So any point of the ship that should follow a stable equilibrium, any disturbances regarding that, that should not capsize, they cannot have a overturning moment to capsize that.

Instead of that it can come to a stable locations, comes to the stable locations. So that is the reasons there are lot of detailed studies there in the people who are designed the ships. But here we will talk about very basic things that how we can find out the stability of a floating object and the stable in a three ways we define natural equilibrium, stable equilibrium, and unstable equilibrium.

Before doing that, let me introduce one point, which is we call the metacenters, okay. That is what is called the metacenter. What is that metacenters, okay? If I tilt it, a floating object to a angle of delta theta, then there will be a new waterlines will come it. That means the shape will change it like the for example, for this case, the will tilting this part. So new waterlines will come it.

And because of that, the buoyancy, the center of buoyancy will change it from B to $B^{'}$. It will change from B to $B^{'}$ and the along that you will have the buoyancy force what is going to act it. The center of buoyancy will change from B to $B^{'}$and along that locations we will have the buoyancy force will act it. If a very simple case which is theoretical case it does not happen in real ship design case.

Because if there is a conditions that the buoyant force and the weight they are acting on the same points, the buoyant force and if I extend the buoyant force, that what this line of the buoyant force is coincide with the CG of the floating object where the weight is acting it. Then that is the point we call metacenters.

That means a vertical line drawn upward by v intersect the line of symmetry at a point which is defined is a metacentric which is a independent θ angle for the small changes. So if a conditions what you have that M is coincide with G, that is what I say the very rare conditions happens that the M the and G will be the same point, the metacenters and the center of gravity in a same point then we call natural equilibrium okay.

That means if you tilt it that again this what will be remain in that shape okay. That what will be a natural equilibrium conditions okay. But many of the cases what it happens is like this called a stable equilibrium. Like we tilted this floating object. As we tilted this floating object, the B is shifted to B'.

The center of buoyancy is shifted from B dash as the waterline changes the displaced water liquid that what is changes and their center of buoyancy changes from B to B'.and FB the buoyant force does not change much, does not change it because the same volume of liquid displayed by this object only the center of buoyancy changes from the B to B'.

Because of that, if you find out the metacentric point, which is M here, and G is the CG point, so in this case, we have a the BM is getting then BG. BM is greater than BG. That means the M is the lies above of the G points. At that time if you look it there will be a restoring moment will be generated, there will be a restoring moment will be generated to bring this the tilted object to the right position, the initial positions.

That is the reasons restoring couple or moment will happen it that will bring back this floating object with a minor tilting again it can come back it. So since this is the process happens where BM is greater than BG and we call it stable equilibrium, we call stable equilibrium. That means this type of ships or any floating object, if you do a minor instability or minor angular moment after certain times, you will see that it again coming back to the initial conditions.

So that type of floating object we call it is at stable equilibrium. Other way round if you have a BM less than BG same concept, okay. Now B has changed to B', the center of buoyancy has changed it, but the M lies in between B and G. But if you look in the moment what generate because of this, the weight and the v will be the overturning moment.

Because of that, this floating body will be capsized. It will be capsized. So in that case the theoretically BM the distance between the buoyancy and the metacentric height, metacenters that BM will be lesser than the BG and we have an unstable equilibrium. So let me summarize that, that when you have a floating object to test its stability, we generate a small disturbance of tilted with a small angle $\Delta\theta$.

And find out because of that, how the center of buoyancy changes it. And that the line of actions of the buoyancy force of new locations, they meet with the vertical axis that is the location is called the metacenter. Thus the location of the metacenter with respect to the CG and the earlier buoyant centers that what will decide it whether it is a natural equilibrium, stable equilibrium, or unstable equilibrium. The theoretically if the BM and BG are having the relationship like BM is greater than BG or BM is less than BG we have a stable or unstable equilibrium.

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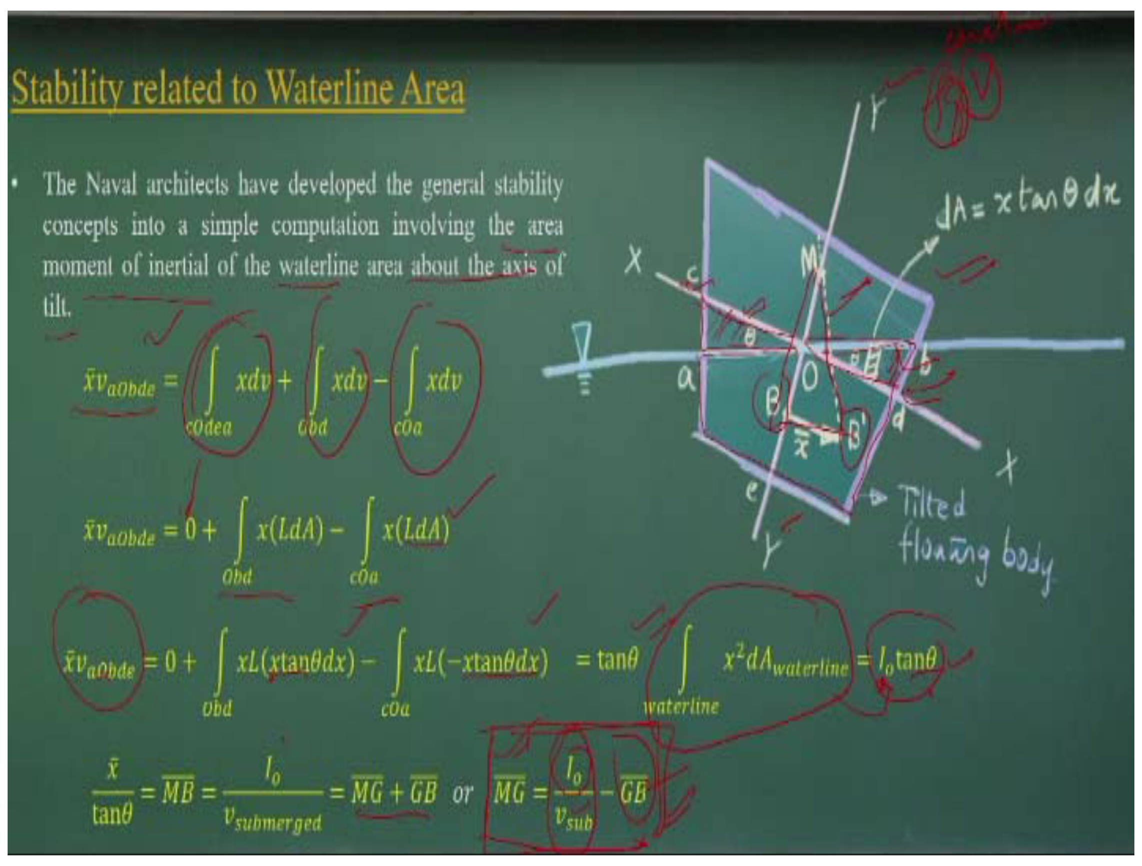

Now how to compute this the metacentric height? Let you have a floating object like this, okay? And you consider the unit width of this ones which is a perpendicular to this surface that is what unit width is there. So what we will do it let have the initial positions of waterlines goes through this C to D. So initially it was C to D and it was tilted okay.

Because of the tilting as I explained earlier the B changes to $B_1^{'}$, B changes to $B_1^{'}$, okay. The basic idea is to find out how we can establish a relationship between the metacentric height okay. So if you look it that the center of buoyancy changes from B to $B^{'}$. That is what is $\bar{x}$distance from this okay. And because of that, what it happens it this area will be exposed out of the liquid.

This is the area come into the liquid, come into the liquid surface. Now we want to compute it, what will be the $\bar{x}$. That what we can do in very simple way that we can take it, the area of the moment of inertia of waterline can be about axis of tilting, about the axis of tilting. That means, YY, if I take it, the force into distance that what is the moment.

But here I am just giving the volume, because the force will be the ρ g and the volume. What will be the force, the weight of the fluid. That what will be the ρ g into volume. Since the ρ g is a constant. So we use just the volume expressions here to equate the moment, nothing else. That is what you have to understand it is a just taking a moment of the force. That is what it act because of this fluid displacement.

$$\bar{x}v_{aObde} = \int_{cOdea} xdv + \int_{Obd} xdv - \int_{cOa} xdv$$

If you look it the first, the moment of this part, volume part we have considered aOBde that means this part, okay? That is what will be the three components will be there. Because of this, this component will be the minus, this component will be positive. That is what is happened. cOBd will be the positive and this way. Since this is the axis of symmetry, these values becomes zero.

$$\bar{x}v_{aObde} = 0 + \int_{Obd} x(LdA) - \int_{cOa} x(LdA)$$

$$\bar{x}v_{aObde} = 0 + \int_{Obd} xL(x\tan\theta dx) - \int_{cOa} xL(-x\tan\theta dx)$$

$$= \tan\theta \int_{waterline} x^2 dA_{waterline} = I_o \tan\theta$$

If you take a $\tan\theta$ common out of that, that means this is what is showing that along this waterline, the dA area the x^2 one that what the moment of inertia along the axis passing through the symmetrical axis that what into the $\tan\theta$. So, So if it is that from the simple geometry you can find that,

$$\frac{\bar{x}}{tan\theta} = \overline{MB} = \frac{I_o}{v_{submerged}} = \overline{MG} + \overline{GB} \quad or \quad \overline{MG} = \frac{I_o}{v_{sub}} - \overline{GB}$$

If you have a look at these triangles, we can find out that this is what by this definition I nut by v submerged the MB will be the MG plus GB. The finally the MG will be this component minus GB. So any object we can find out the center of gravity.

You can find out the center of buoyancy. That what can be computed. We can find the v submerged. We can get it I nut then you can compute the MG. If MG is positive then it is a stable equilibrium, okay. If MG equal to zero, then we can say it is a natural equilibrium, okay. It MG is negative, then we call unstable equilibrium conditions. So with this, we compute it that what will be the MG of a floating object.

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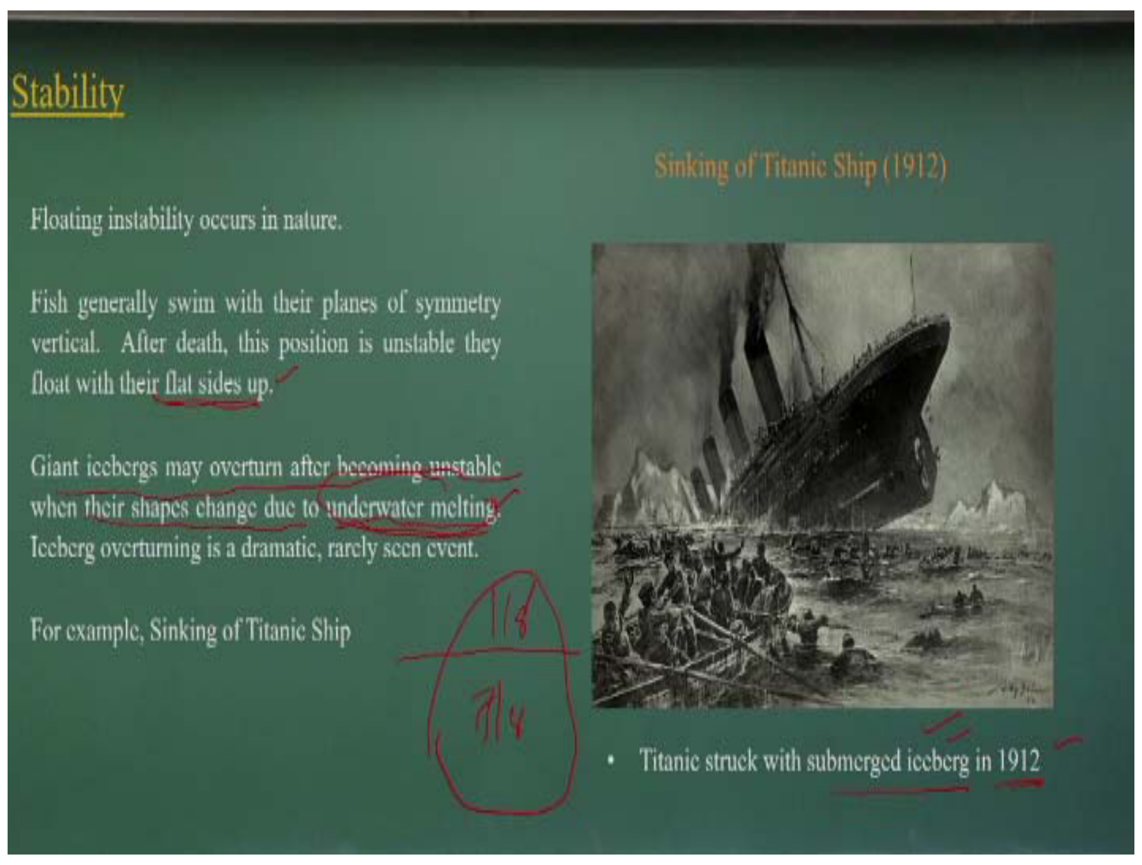

Just let me see this just want to say that what are the applications of the floating, stability of the floating objects okay. And if you look at that if you if we can see observe of a dead fish you can see that it is what tilted in when a fish is dead conditions. So finally you see that it is floating as a side unstable, with flat sides up.

This because of the change of the center of buoyancy and when the fish is swimming the symmetry is different and once it is dead that what it changes. It cannot be stability position of that. And many of the interesting photographs or the video you can see it that the melting of icebergs or falling of the icebergs, okay. Very beautiful, scenic beauty has come it.